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PLANAR CIRCUIT EQUATION and its PRACTICAL APPLICATION to PLANAR-TYPE TRANSMISSION-LINE CIRCUIT

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ABSTRACT

This paper presents general circuit equation of planar circuit basing on the field theory and systematic method of analysis basing on the conventional circuit theory. As an example of the application, planar-type transmission-line circuits are analyzed.

Introduction

Planar circuit whose concept was proposed by Okoshi in 1969 is now considered to be important from practical and theoretical point of view[1]-[4]. That is

1. Planar circuit has a strong posibility to be used widely as a part of integrated circuit at frequency range from microwave to optical frequency.
2. Lumped-element circuit(0-dimension in space) —Transmission-Line circuit(1-dimension) —Planar circuit(2-dimension) —Bulk circuit(3-dimension). This is a natural process in the development of circuit theory. As for the former two(0-dimension and 1-dimension) their circuit equations are clear; systematic method of analysis and synthesis have been established. Hence the next is 2-dimensional circuit.

After the proposal of Okoshi, many works as for analysis, synthesis and structure of planar circuit have been done. However these works were individual and carried out mainly depending on the field theory; they are lacking in general view points of circuit theory. Hence now is the time to formulate circuit equation for planar circuit and establish systematic method of the analysis and the synthesis.

General Planar Circuit Equation

An arbitrary-shaped planar circuit is shown in Fig.1, where Voltage $V(x,y)$ and 2-dimensional vector current $\mathbb{J}(x,y)$ are assumed to be defined properly.

Then the planar circuit equations are given by the following two equations.

$$\text{grad } V(x,y) = -Z\mathbb{J}(x,y)\ ,\ \text{div } \mathbb{J}(x,y) = -YV(x,y)$$

where grad and div are two dimensional operators; Z and Y are series impedance and shunt admittance respectively, which are exactly defined and derived from Maxwell's equations as shown in table 1.

Two Types of Mode

As shown in Fig.2, there are two types of field distribution in the planar circuit. One is TM-type and the other is TE-type. For the TM type, magnetic field is parallel to the plane; electric voltage and electric current are defined. For the TE-type, electric field is parallel; magnetic voltage and magnetic current are defined. Of course there exists an infinite number of height-mode for each type. Usually the lowest height-mode is used.

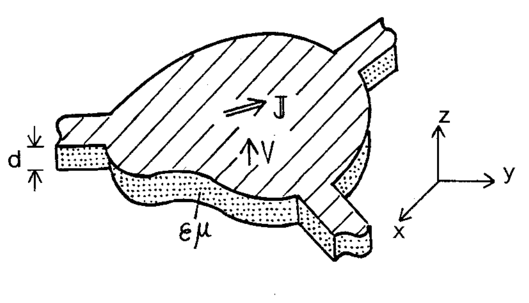

Fig.1 Planar Circuit.

Circuit Theory basing on Mode Impedance

Usually planar-type transmission lines such as shown in Fig.3 are connected to the planar circuit. Different height-mode from that of planar circuit can not be excited in the transmission-line because of the same height with the planar circuit and no discontinuity in the height direction.

However width-modes in the transmission line are excited, principally in number, at the junctions with planar circuit. In order to take into account these width-modes following mode impedance(between p-th width-mode in i-th transmission-line and q-th width-mode in j-th transmission-line) of planar circuit is defined.

$$Z_{p;q}^{i;j} = Z\int_0^{W(i)}\int_0^{W(j)} f_p^{(i)}(s^{(i)})\, G(s^{(i)},s^{(j)})\, f_q^{(j)}(s^{(j)})\, \frac{ds^{(i)}}{W(i)}\frac{ds^{(j)}}{W(j)}$$

where G is the scalar Green's function of the two dimensional Helmholtz equation with proper boundary condition; f is the p-th width-mode function in the i-th transmission line. Using mode impedance input/output characteristics of planar circuit can be calculated basing on the conventional circuit theory.

Problems of the Above Circuit Theory

When the above circuit theory is applied to the analysis of planar circuit, following problems arise.

1. How to calculate the Green's function of an arbitrary-shaped planar circut.
2. How many width-modes in transmission-line must be taken into account for the calculation of input/output characteristics.

As for the problem 1, Green's function can be developed as follows.

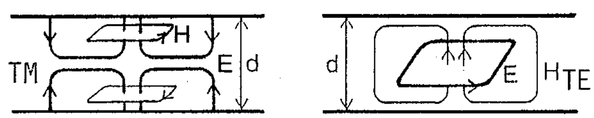

Fig.2 Field Distribution of TM and TE-Type

	TM(E) mode	TE(H) mode
Field Theory	$\mathbb{E}=(\mathbb{E}_t, E_z)$	$\mathbb{E}=(\mathbb{E}_t, 0)$
	$\mathbb{H}=(\mathbb{H}_t, 0)$	$\mathbb{H}=(\mathbb{H}_t, H_z)$
	$E_z(x,y,z)=E_z(x,y)\,COS(\frac{n\pi}{d}z)$	$H_z(x,y,z)=H_z(x,y)\,SIN(\frac{n\pi}{d}z)$
	$\mathbb{H}_t(x,y,z)=\mathbb{H}_t(x,y)\,COS(\frac{n\pi}{d}z)$	$\mathbb{H}_t(x,y,z)=\mathbb{H}_t(x,y)\,COS(\frac{n\pi}{d}z)$
	$\mathbb{E}_t(x,y,z)=\mathbb{E}_t(x,y)\,SIN(\frac{n\pi}{d}z)$	$\mathbb{E}_t(x,y,z)=\mathbb{E}_t(x,y)\,SIN(\frac{n\pi}{d}z)$
	$(n=0,1,2,3,----)$	$(n=1,2,3,4,----)$
Circuit Theory	$V^E=-E_z(x,y)d$ (V)	$V^H=-H_z(x,y)d$ (A)
	$\mathbb{J}^E=\mathbb{H}_t(x,y)\times \mathbb{k}$ (A/m)	$\mathbb{J}^H=\mathbb{k}\times\mathbb{E}_t(x,y)$ (V/m)
	$Z^E=j\frac{\beta_t^2}{\omega\varepsilon}d$ (Ω)	$Z^H=j\frac{\beta_t^2}{\omega\mu_0}d$ (S)
	$Y^E=j\omega\varepsilon d$ (S)	$Y^H=j\omega\mu_0 d$ (Ω)
	$\beta_t=\sqrt{\omega^2\varepsilon_0\mu_0\varepsilon_S-(\frac{n\pi}{d})^2}$	$\beta_t=\sqrt{\omega^2\varepsilon_0\mu_0\varepsilon_S-(\frac{n\pi}{d})^2}$
	$Z_c^E=\frac{\beta_t}{\omega\varepsilon}$ (Ω)	$Z_c^H=\frac{\beta_t}{\omega\mu_0}$ (S)

Table 1 TM and TE-type Voltage and Current.

$$G(x,y;x_0,y_0)= -\sum_n \frac{1}{(\beta_t^2 - k_n^2)S} \varphi_n(x_0,y_0)\, \varphi_n(x,y)$$

where φ_n and k_n are normalized normal mode function and eigenvalue of the planar circuit. The calculation of these normal modes is not difficult because of the recent development of high-speed computer and good algorithm. The essential problem of this method is the error caused by the truncation of modes in the planar circuit.

However this error can be reduced to any degree by taking into account the necessary number of modes in the planar circuit. As for problem 2, the error can be again reduced to any degree by the same way as the problem 1, because higher width-mode is more evanescent. Thus systematic analysis of an arbitrary-shaped planar circuit becomes possible by normal mode analysis.

Practical Application of the Theory to Planar-Type Transmission-Line Circuit

It is worth while to show that circuit theory of planar circuit is practically useful for the analysis and design of planar-type transmission-line circuit. At this case the effect of discontinuity at junction of transmission-line can be automatically taken into account by this theory.

Frequency characteristics of the following circuit were calculated in this paper. The results are shown in Fig.4-11.

TRANSMISSION LINE	HIGHT-MODE	3-dB HYBRID CIRCUIT	CIRCULAR RIGHT ANGLE BEND	CORNER CUT RIGHT ANGLE BEND
RECTANGULAR WAVEGUIDE	TM	NOT-YET	Fig.6	Fig.9
	TE	Fig.4	Fig.7	Fig.10
TRIPLATE-LINE (STRIP-LINE)	TM	Fig.5	Fig.8	Fig.11

Conclusion

General planar circuit equations are formulated basing on Maxwell's equations and systematic method of the analysis are established. Existence of TE and TM-type planar circuit is also pointed out.

As an example of practical application of the theory, planar-type transmission-line circuit such as strip line and rectangular waveguide circuit are analyzed. Practical application of planar circuit in the true sense and general synthesis method of planar circuit are left as future problems.

ACKNOWLEDGMENT

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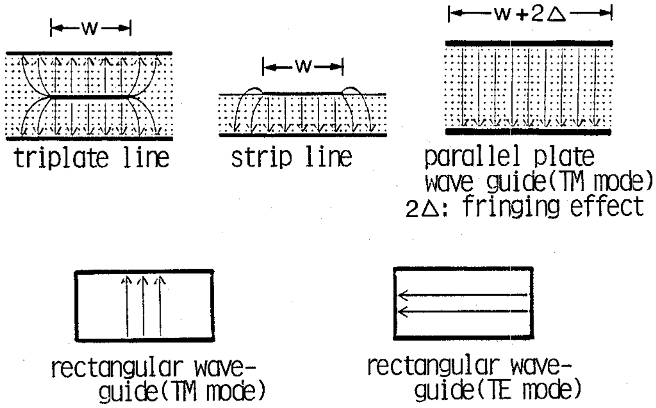

Fig.3 Example of Planar-type Transmission Line

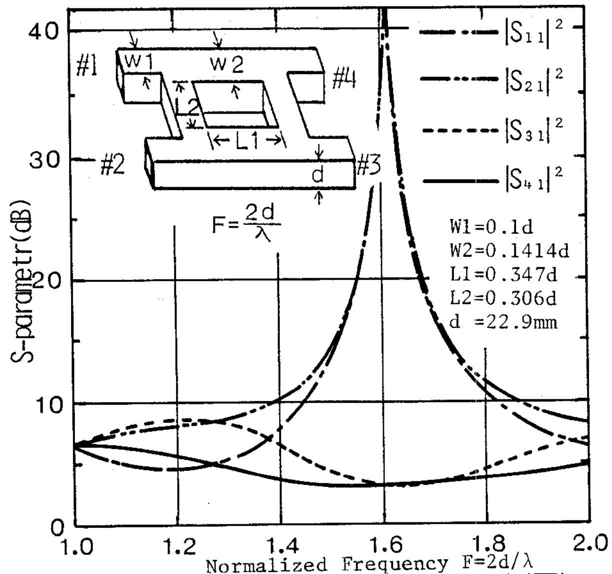

Fig.4 3-dB Hybrid of Rectangular Waveguide(TE)

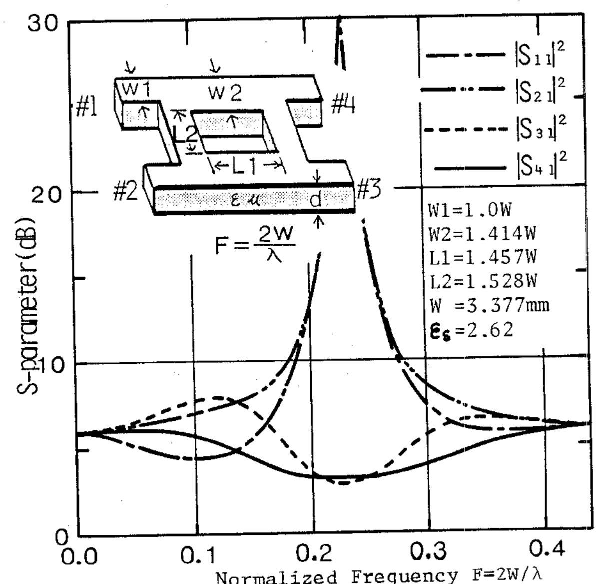

Fig.5 3-dB Hybrid Strip LINE(TM)

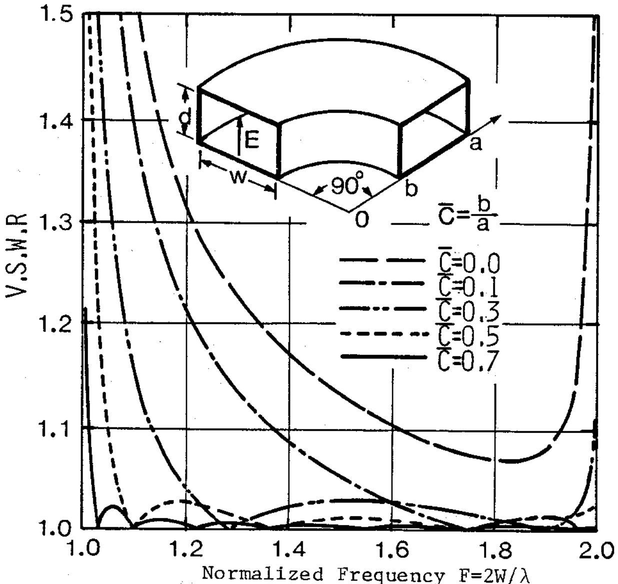

Fig.6 Circular Bend of Rectangular Waveguide(TM)

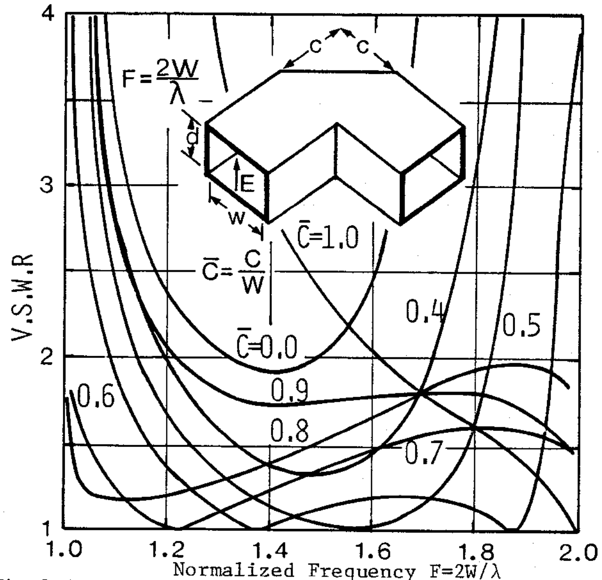

Fig.9 Corner-Cut Bend of Rectangular Waveguide(TM)

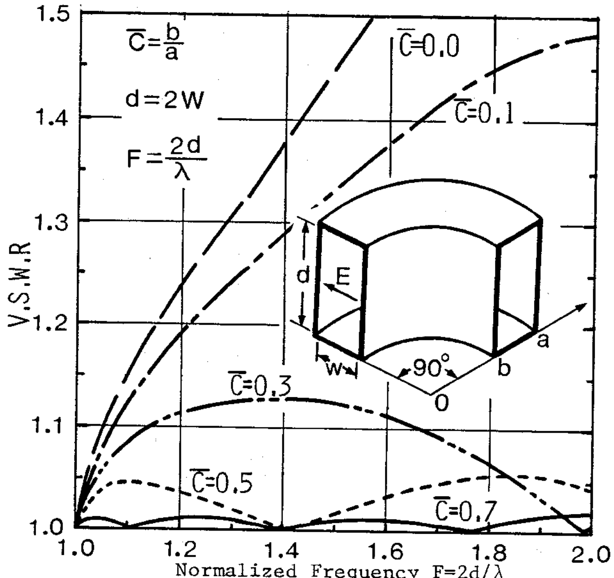

Fig.7 Circular Bend of Rectangular Waveguide(TE)

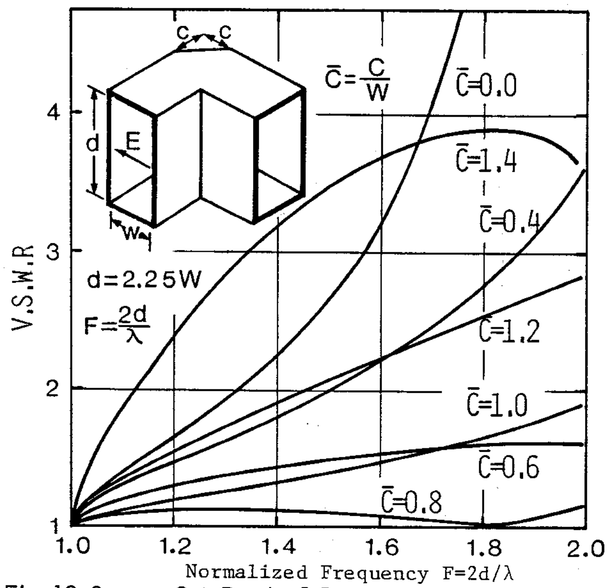

Fig.10 Corner-Cut Bend of Rectangular Wavegide(TE)

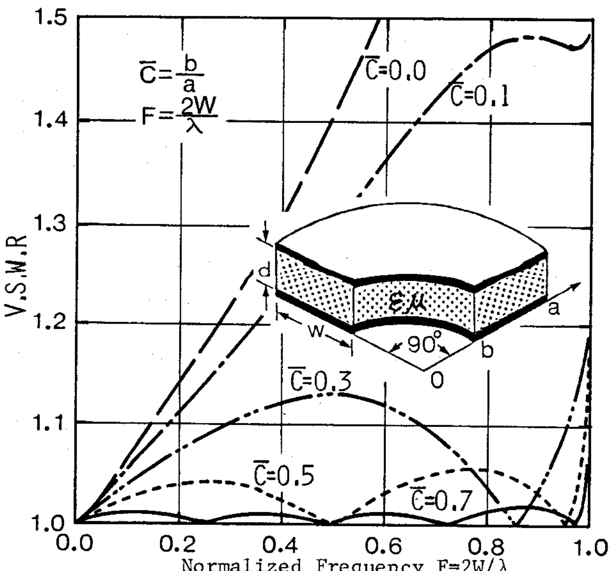

Fig.8 Circular 90°Bend of Strip Line(TM)

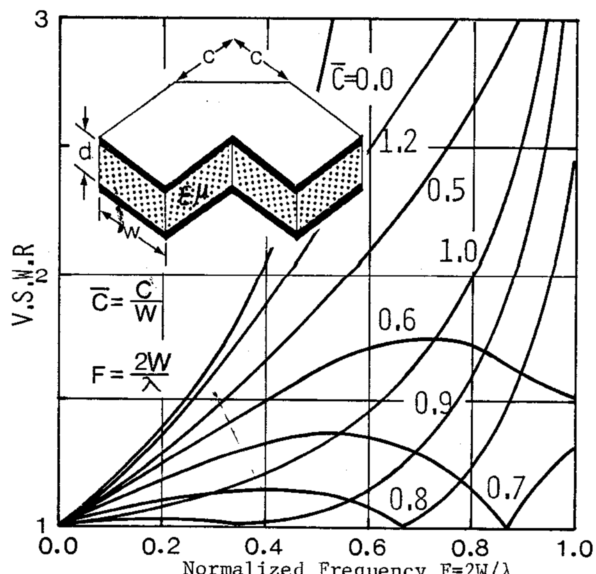

Fig.11 Corner-Cut 90°Bend of Strip Line(TM)